Rigid-Foldable Thick Origami

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Abstract

In this paper, a method is proposed for geometrically constructing thick panel structures that follow the kinetic behavior of rigid origami by using tapered or two-ply panels and hinges located at their edges. The proposed method can convert generalized pattern of rigid-foldable origami into thick panels structure with kinetic motion, which leads to novel designs of origami for various engineering purposes including architecture.

1 Introduction

Rigid-foldable origami or rigid origami is a piecewise linear origami that is continuously transformable without the deformation of each facet. Therefore, rigid origami realizes a deployment mechanism with stiff panels and hinges, which has advantages for various engineering purposes, especially for designs of kinetic architecture. In a mathematical context, origami is regarded as an ideal zero-thickness surface. However, this is no longer true when we physically implement the mechanism. Especially when we utilize the stiffness of panels for large-scale kinetic structures, it is necessary to consider a mechanism that allows thick panels. For example, in the design of architectural space, we need structures composed of thick panels or composite three-dimensional structure with finite volume in order to bear the gravity and the other loads and to insulate heat, radiation, sound, etc.

Thick panels origami with symmetric degree-4 vertices have been proposed using shifted axis such as [Hoberman 88] and [Trautz and Künstler 09]. However, no method that enables the thickening of freely designed rigid origami was proposed; such a freeform rigid origami can be obtained as a triangular mesh origami or a generalized rigid-foldable quadrilateral mesh origami [Tachi 09a]. This paper proposes a novel geometric method for implementing a general rigid-foldable origami as a structure composed of tapered or non-tapered constant-thickness thick plates and hinges without changing the mechanical behavior from that of the ideal rigid origami. Since we can obtain the valid pattern for a given rigid-origami mechanism, the method can contribute to improving the designability of rigid-foldable structures.

2 Problem Description

In this section, we overview the problem of thickening origami, and show existing approaches tackling this problem. The simplest thick rigid origami structure is a door hinge, which is a thick interpretation of single line fold. In this case, the rotational axis is located on the valley side of the foldline. Here we call this type of approach *axis-shift* since the axis is shifted to the valley side of the thick panel. Axis-shift can also convert a corrugated surface without interior vertex such as repeating mountain and valley pattern to a folding screen composed of thick plate mechanism. This type of structure can fold and unfold completely from 0 to π. However, axis-shift method is not always successful for typical rigid origami mechanism with interior vertices. This problem can be described as follows.

2.1 Rigid Origami without Thickness

First, we illustrate the kinematics of ideal ideal, i.e., non-thick, rigid origami. The configuration of rigid origami is represented by the folding angles of its foldlines, which are constrained around interior vertices. This constraint can be represented as the identity of rotational matrix as used by Belcastro and Hull [Belcastro and Hull 02], Balkcom [Balkcom 02], and Tachi [Tachi 09b]. This essentially produces 3 degrees of constraints for each interior vertex that fundamentally correspond to the rotations in x, y, and z direction of facets around the point of intersection of incident foldlines. As a result, a rigid origami produces a kinetic motion where foldlines fold simultaneously. Since the number of vertices, facets and edges are related by the Euler characteristic of the surface, which is 1, the degrees of freedom of overall system is limited. Specifically, a model has at most $N_0 - 3$ degrees of freedom (assumed that all facets are triangulated), where N_0 is the number of vertices on the boundary of the surface. Especially, in the case of quadrilateral-mesh based origami, such as Miura-ori, the number of foldlines is smaller than the number of constraints. This produces either an overconstrained structure without kinetic motion or a 1 DOF kinetic structure with redundant constraints; the condition for a quadrilateral mesh origami to have kinetic motion is investigated in [Tachi 09a] to allow freeform generalization of Miura-ori.

In the context of utilizing the kinetic behavior of general origami, the axis-shift approach has a problem since the typical kinetic behavior of origami that every edge folds simultaneously is produced by the interior vertex that constrains the folding motion. In the case of thick origami with axis-shift, an interior verticex generally produces 6 constraints throughout the transformation (3 rotation and 3 translation) since the foldlines are not concurrent anymore. This normally produces overconstrained system, where no continuous motion can be achieved. Even if we succeeded in designing the consistent pattern in finite number of states, this produces multi-stable structure without rigid-foldabilty.

2.2 Existing Methods

A few approaches have been proposed in order to solve the problem of thickening.

Symmetric Miura-ori Vertex Hoberman [Hoberman 88] designed a degree-4 vertex by thick panels that connects shifted axes of rotation using plates with two level of thicknesses. This gives a structure that enables a one-DOF folding motion between completely unfolded and folded states represented by rotation angle (0 to π) (Figure 1). The structure can be applied for designing Miura-ori or Miura-ori based cylindrical sirface. is proposed.

The most significant limitation of this structure is that it cannot be applied for non-symmetric or non flat-foldable vertices, e.g., a variational design of flat-foldable degree-4 vertex thickened with this approach forms a bistable structure where the connectivity breaks unless it is completely unfolded or folded. In fact, the application is only limited to symmetric vertex of Miura-or, this only enables one parameter variation. Another notable limitation of this approach is that it cannot allow multiple overlap of plates. In a case where alternately adjacent facets, i.e., sharing the same adjacent facet, overlap in the folded state, the panel of shared facet is separated into two as the half-thickness volume of overlapped part is removed in this approach.

Slidable Hinges An implementation method by slidable hinges is proposed by Trautz and Künstler [Trautz and Künstler 09]. This method adds extra degrees of freedom by allowing the foldlines to be slided along the rotational axes. The number of variables is doubled by such slidable hinges to compensate the doubled number of constraints around each vertex. They have shown thick panel kinetic structures with symmetric degree-4 vertices that can be folded to $\pi - \delta$ value where δ relates to the amount of slide. Since the sliding amount of an edge is shared by adjacent vertices, the behavior is determined globally for a general case, although the global behavior of slidable hinges structures have not been sufficiently analyzed. In fact, this global behavior can be a critical problem for some patterns,

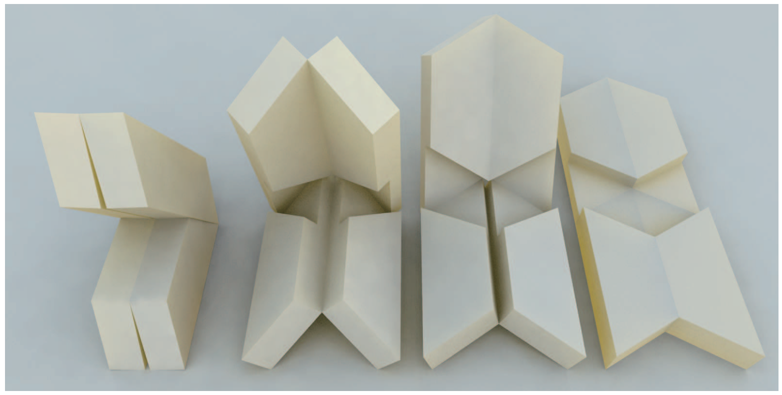


Figure 1: The folding motion of thickened symmetric degree-4 vertex.

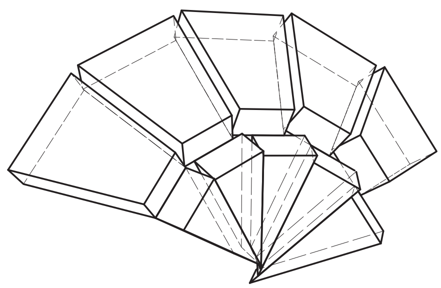
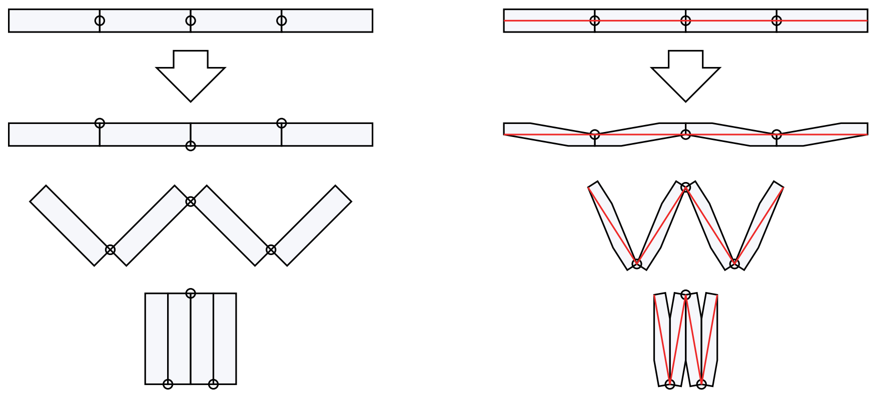


Figure 2: An example of slidable hinges where the sliding value is accumulated at the hinges on the right.

i.e., we can easily show an example that this model fails (Figure 2), where the sliding value is accumulated at one of the edges to produce separation or intersection of volumes. Therefore, slidable hinges do not allow direct interpretation of general origami.

3 Proposing Method

Tapered Panels In order to enable the construction of generalized rigid-foldable structure with thick panels, we propose kinetic structures that precisely follow the motion of ideal rigid origami with zero thickness (Figure 3(b)) by locating the rotational axes to lie exactly on the edges of ideal origami. This has a great advantage over previous axis-shift approaches



(a) (b)

Figure 3: Two approaches for enabling thick panel origami. (a) Axis-shift. (b) The proposed method based on trimming by bisecting planes. Red path represents the ideal origami without thickness.

(Figure 3(a)) that the folding motion is estimated only by the kinematics of ideal origami.

The procedure of creating thick panels is as follows. First, a zero-thickness ideal origami in the developed state is first thickened by offsetting the surface by constant distance in two directions; in this state, the solids of adjacent facets collide when the origami tries to fold. Then the solid of each facet is trimmed by the bisecting planes of dihedral angles between adjacent facets (Figure 4) in order to avoid the collision of volumes. The shape of the solid changes according to the folding angles of edges. By first assuming the maximum and minimum folding angles that the thick origami can fold, we can obtain the solid that works within that range. Since half of the volume of the solid becomes zero when the maximum folding angle of an edge equals to π, we cannot completely flat fold the model. Thus we use $\pi - \delta$ for the maximum folding angles. Now the structure follows the kinetic motion of rigid origami without thickness because all foldlines are located on the center of the panel, i.e., the ideal origami surface.

The upper bound of each folding angle $\pi - \delta$ is determined by thickness of the panels. If we project a solid facet onto a plane, an edge on the top facet is an offset of the original edge by the distance of $t \cot \frac{\delta}{2}$, where t is the thickness of the panel and the maximum folding angle of the edge is given by $\pi - \delta$, respectively. The intersection of adjacent offset edges determines the corresponding corner points on the offset volume. This

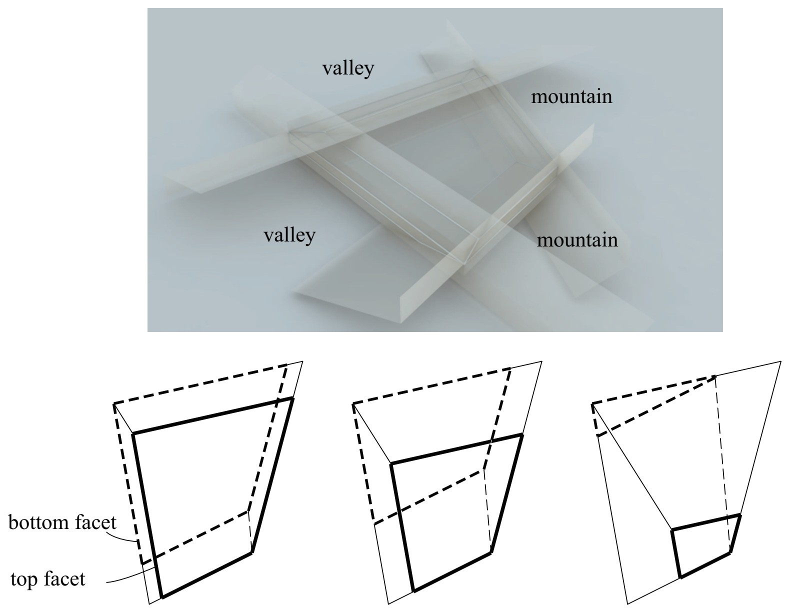


Figure 4: Trimming the volume by bisecting planes of dihedral angles between adjacent facets.

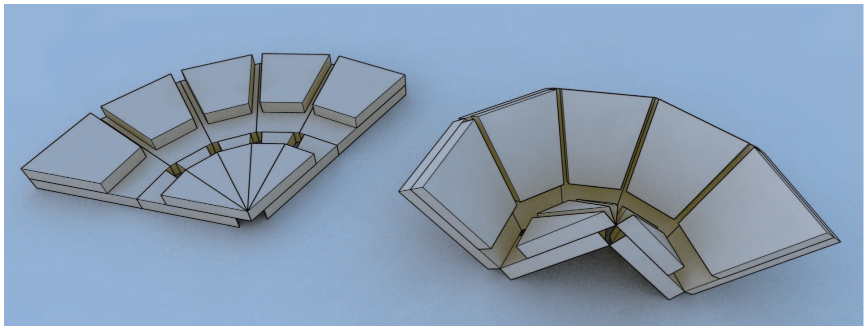


Figure 5: A model with constant thickness panels. Notice the difference with slidable hinge method as shown in Figure 2.

is similar to calculating skeleton of the polyhedron, however we stop this process when two offset corners are merged into one to keep the shape of the top facet. Therefore the amount of possible offset is limited by the size of the panel, and thus dihedral angle δ is related to the thickness of the panel as $\tan\frac{\delta}{2} \propto t$. If we try to thicken the panel, the packaging efficiency of the structure lowers.

Constant Thickness Panels If the thickness-width ratio for each panel is small enough compared to incident minimum dihedral angles, so that the top and bottom facets share a significant amount of area in a top view, the tapered solid can be substituted by two-ply constant thickness panels. In the case of constant thickness panels, the structure can be easily manufactured via a simple 2-axis cutting machine; this significantly simplifies cutting procedure while produces holes at corners of panels. Figure 5 shows the folding motion of an example model with constant thickness panels.

Global Collision In our proposing method, we have assumed that the collision between thick panels occurs only at the foldline. Even though this approximation works for many models, this is not true in a general sense since there can occur global collisions. In order to avoid global collision between non adjacent panels, we can naturally extend the proposed method: calculate the bisecting plane for each pair of intersecting facets and cut out the volume of panels along the plane, or precisely, the swept plane to allow continuous motion.

Characteristics Also, since any foldline cannot fold up completely to π, we cannot produce a folding mechanism with singular two motions such

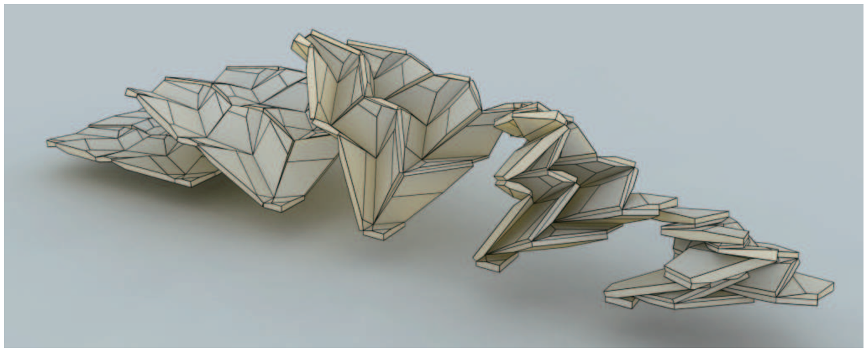


Figure 6: A quadrilateral-mesh hypar model with thick tapered panels.

as a vertex with four $\pi/2$ corners. This is a disadvantage of our method since axis-shift of symmetric Miura-ori vertex can produce singular motion. Therefore our method is most suitable for producing mechanisms with simultaneous folding motions.

4 Application for Designs

The proposed thickening method is implemented as a parametric design system using *Grasshopper* [McNeel] and VC# script. This successfully yielded a rigid-foldable structure with thickness producing the identical mechanism as the ideal rigid origami. The connection part can be realized as embedded mechanical hinges whose rotational axes are located exactly on the ideal edges. Also, non-mechanical hinges can be constructed by sandwiching a strong fabric or film between the two panels since rotational axes are located on the center plane.

A realized example design of constant thickness rigid origami composed of quadrilateral panels is shown in Figure 8. This 2.5 m $\times$ 2.5 m square model is manufactured from two layers of double-walled cardboards (each of which is 10 mm thickness) sandwiched by a cloth. Because of its one-DOF mechanism, a simultaneous motion that counterbalances the weight is produced. This enabled a smooth and dynamic motion by lightly pushing the rim of the structure. A prospective design possibility is in applying the method for kinetic architectures (Figure 9).

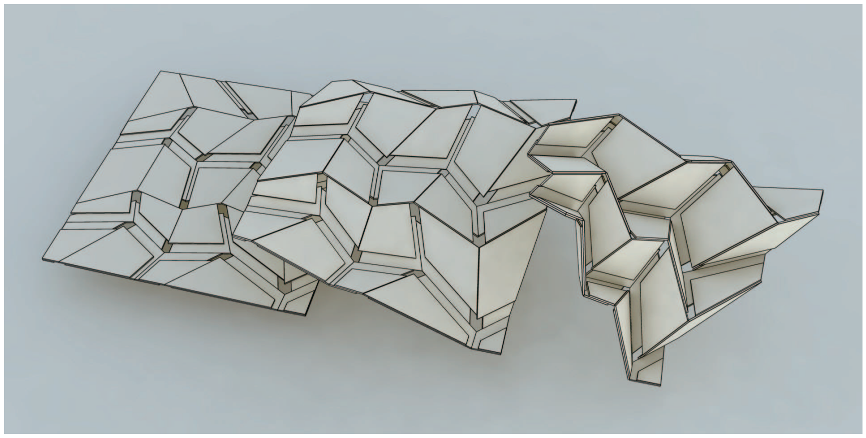


Figure 7: Volume substituted by two constant-thickness panels.

Figure 8: An example design of rigid foldable origami materialized with cloth and cardboards.

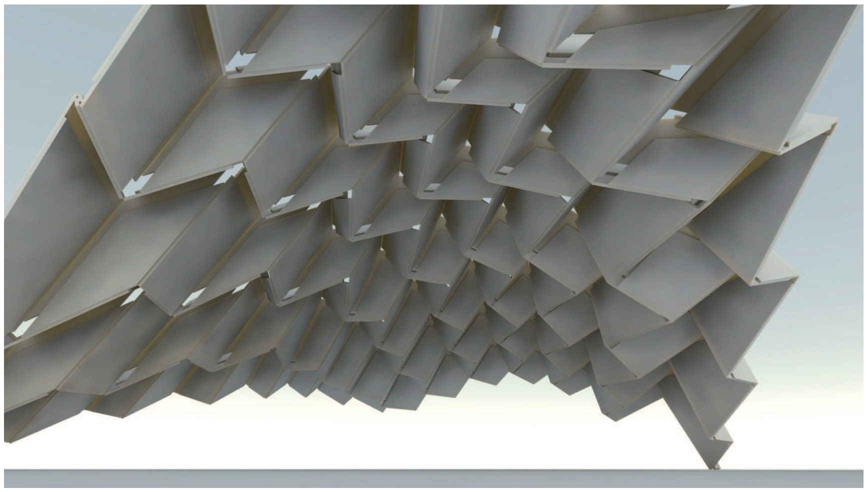


Figure 9: An image of architectural-scaled rigid origami.

5 Conclusion

This paper presented a novel method for enabling a rigid-foldable origami structure with thick panels while preserving the kinetic behavior of ideal origami surface. The method trims intersecting part between panels and produces a kinetic mechanism that folds between predefined minimum and maximum folding angles. The maximum folding angle $\pi - \delta$ and the thickness of panels t are related by $\tan \frac{\delta}{2} \propto t$. Our method successfully produced rigid origami designs applicable for human scale structures.

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