Rigid Origami Structures with Vacuumatics: Geometric Considerations

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Summary

We propose a novel type of hybrid structures, in which multi-DOF rigid foldable structures will be stiffened using Vacuumatics i.e., the structural system of negative pressured double membrane containing particles. The structure is transformed to a desired 3D configuration following the kinematics of origami when the hinges are plastic and flexible, and then stiffened at dihedral hinges by strengthening negative pressure. In addition, the negative pressure gives a small moment at the hinge lines, which helps to solve singularity problem of rigid origami at the initial unfolded state.

Keywords: origami; vacuumatics; rigid-foldable structures.

1. Introduction

Rigid foldable structures based on rigid panels and hinges, i.e., rigid origami, are potentially applicable for creating transformable or temporary spaces. The core advantage of rigid origami is in their geometric scalable mechanisms, i.e., can be scaled up to large scale, and their property of forming continuous surfaces without in-plane deformation. In particular, multi-DOF rigid foldable structures based on triangle panels have an advantage in its flexibility to follow the change in the environment and human activities while the 3D form becomes geometrically stable after fixing the boundary points. One of such example patterns is doubly-expandable transformable shells by Resch and Christiansen [1]. However, such a structure tends to be structurally flexible in reality, even after it is geometrically pinned at its boundary. For example in the case of [1], numbers of rods (3 for each concave vertex) are added in order to stabilize the structure; these fixing rods make the structure not actually transformable or reusable. Therefore, effective temporary stiffening methods have been required.

In order to effectively solve this problem, we utilize another structural system called *Vacuumatics*, which is a negative pressured double membrane containing particles. The structure is initially plastic or viscoelastic, but by adding the negative pressure, the friction force between compressed particles makes the structure stiff. As far as we know, the oldest published concept of an adaptive structure using Vacuumatics is “LIVING-ROOM” by J. Gilbert in 1971 [2 pp.52-53], preceded by simple mockup test of a vault in 1970. Vacuumatics concept is researched by several parties in 1970s, such as by Frei Otto [3]. It is after decades that the structural principle is paid attention again when it is rediscovered by people such as Sobek et al. in 2000s [4] [5].With the social background of sustainability, the potential of Vacuumatics as adaptive insulation also attracts some researchers after the rediscovery. The concept of controlling the stiffness and plasticity of the material by the strength of vacuum has been studied [6]. One of the unsolved problems of pure Vacuumatics system is the method for controlling its overall shapes, which is essential in order to use the concept for adaptive structure.

We propose a novel type of hybrid structures, in which multi-DOF rigid foldable structures are stiffened at their hinges using Vacuumatics. The key concept is that the stiffness of hinges is controlled by the strength of the vacuum asserted to the structure. The structure is transformed to a desired 3D configuration when the hinges are plastic and flexible, and then stiffened at dihedral hinges by strengthening negative pressure. In addition, the negative pressure gives a small moment at the hinge lines at the initial construction, helping to remove the instability caused by the singularity in the unfolded state.

In this paper, we explain this novel concept from the geometric perspective. We will review the kinematics of rigid origami structures to show the singularity in the initial unfolded state and the relation between the 3D form and the 2D boundary configuration.

2. System Overview

2.1 Origami Pattern

Since any rigid origami pattern can be used for the hybrid system, we especially choose a triangle based regular corrugation pattern of origami composed of both convex and concave vertices. Here, a vertex is convex (or concave) when the solid angle around the vertex is greater (or less) than 2π. This includes corrugation patterns by Resch [1], origami patterns known as waterbomb corrugation or "namako" [7], and triangulated version of Miura-ori [8]. It is preferable to include both convex and concave vertices in order to produce a freeform surface having both positive and negative mean curvatures. For example, the PCCP shell or Yoshimura pattern [8] is not suitable for this purpose because it comprises only with convex vertices.

Here, we use a variation of waterbomb corrugation based on 36° grids (Figures 1 and 2). This is compactly flat-foldable and flexible enough in the limited range of folding angle (from 0° to 161° as described in the following subsection).

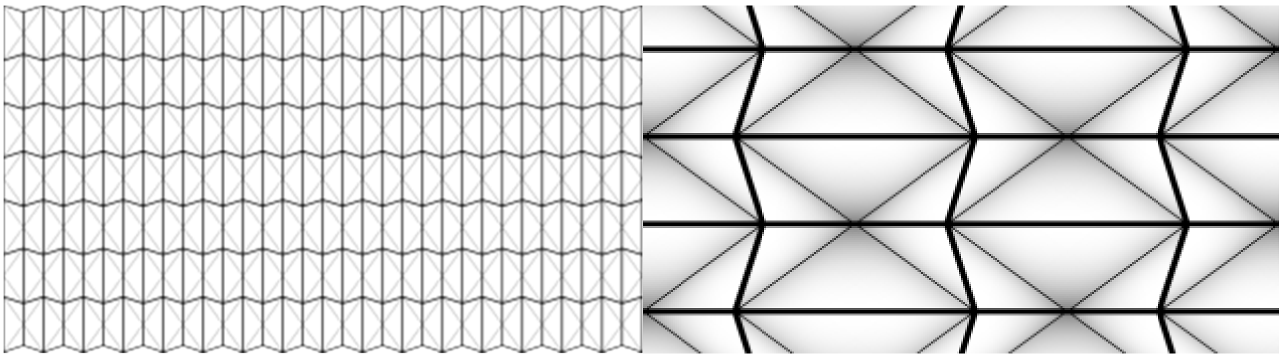

Figure 1: Left: A variational waterbomb corrugation pattern. Right: close-up of the pattern (thick: mountain fold, thin: valley fold). A vertex with 4 (or 2) mountain folds is convex (or concave resp.).

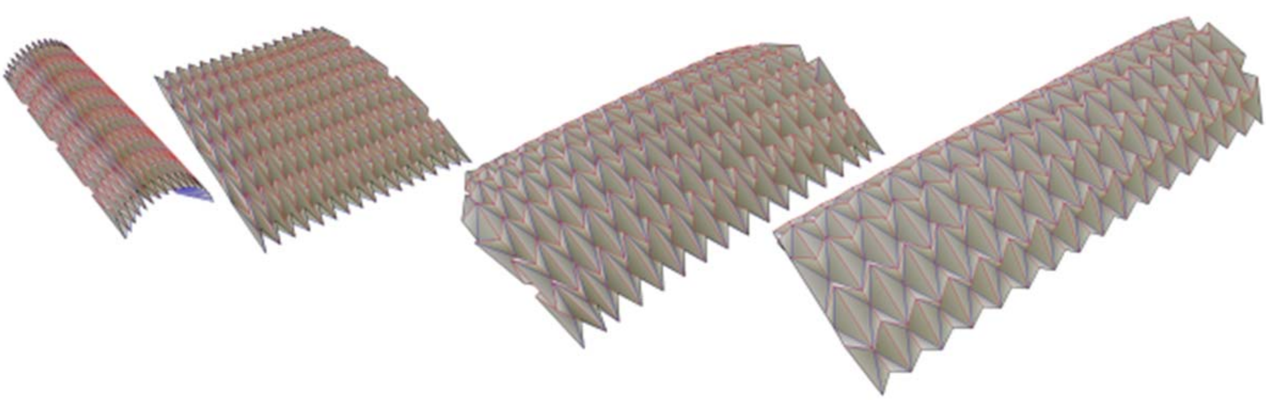

Figure 2: Sequence of unfolding

2.2 Hinge Structure

Flexible hinges allow the continuous rigid plates to be transformed to a desired configuration; however they must be stiffened to hold it against the self-weight and imposed loading. This requirement is essential in the folding process as well as in the final folded state. Our proposed solution is that the flexible hinge is stiffened by given negative pressure.

2.2.1 Layout

Rigid origami structures can be built with double layered thick composite panels. In order to avoid the collision of panels, we offset the panel outline as proposed in [9]. The width of offset is determined by maximum folding angle and the thickness, in the example design, the folding angle can change from 0°to 169°, with the ratio of thickness to the length (longest part) of exterior and interior panel are approximately 0.009 and 0.006 respectively.

This produces gaps between the panels on one of its sides where we fill in aggregate particles packed in breathable material. Then the whole structure is packed in an airtight membrane forming a single air chamber (Figures 3 and 4)

2.2.2 Angular stiffening

The structural principle of vacuum is the difference of the air pressure between inner and outer enclosed volume. Smaller inner air pressure act on the aggregate particle as compression force which is balanced with the tension force loaded on the enclosing membrane. By the compression force the aggregate particles are deformed and their maximum contact between each of them is achieved. Under this prestressed status the stiffened angular hinge can be effective as supporting structures of the global system and could work until the compression stress remains equal to or larger than the tensile stress caused by loading.

Another interesting aspect of Vacuumatics structure is in its plastic behavior that the particles reconfigure themselves to be in another equilibrium state. Therefore, we can produce a plastic hinge whose strength is controlled by the strength of negative pressure. Asserting appropriate amount of (smaller) vacuums keeps the whole structure in a plastic state and assists the process of folding by removing excess force asserted to the control points.

2.2.3 Small additional moment

To allow folding the rigid plates in one direction, the packing particle is set on only one side (valley folding side) of the plates. Consequently the negative pressure gives a small moment at the hinge lines. Certainly the resulting moment would be regarded as an unfavorable effect in the structural point of view, however this can be helpful to solve singularity problem of rigid origami at the initial unfolded state.

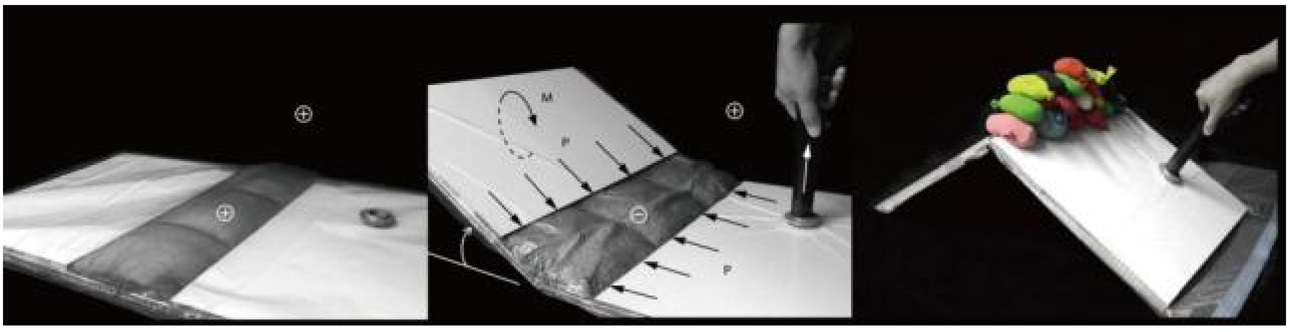

Figure 3: Hinge Design. Left: particles are placed in between panels. Middle: Self-folding moment is induced by the vacuum. Right: The vacuum keeps the hinge stiff.

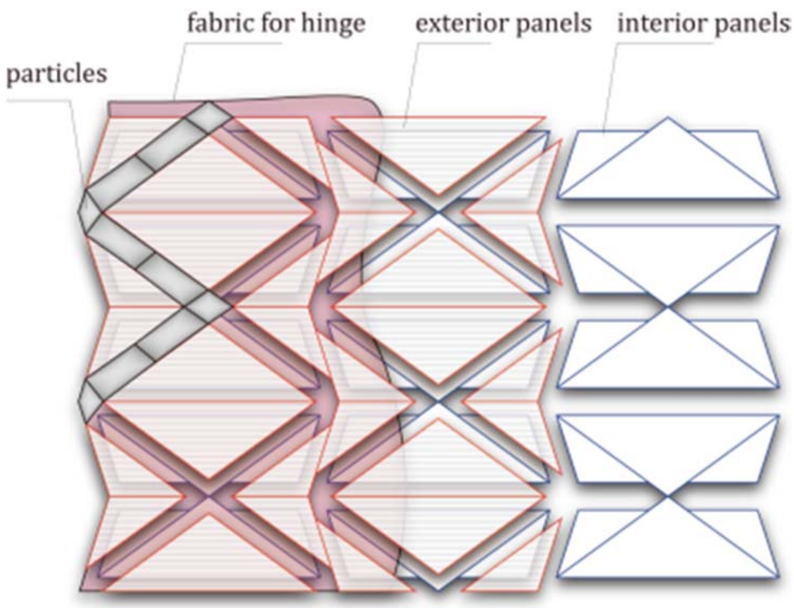

Figure 4: Hinge Design

2.3 Construction Process

The prospective construction process is as follows (Figure 5):

(1) Prefabrication:
Layout the membranes, panels, particles, and fabrics in a plane to form the composite structure. Then, the negative pressure is added to the structure, which induces the small folding moment. This makes the structure non-singular, and the whole folding motion can be actuated by constraining the boundary points. Fold the structure to a compact state so that it can be carried to the site.

(2) On-site deployment:
Unfold the structure and transform it by controlling the configuration of the boundary vertices. The folding process is performed under an appropriate amount of vacuum to make the structure plastically deformable. Strengthen the vacuum when the structure is fixed to the final state. The structure can be re-used by removing the vacuum and re-folding it to the compact state.

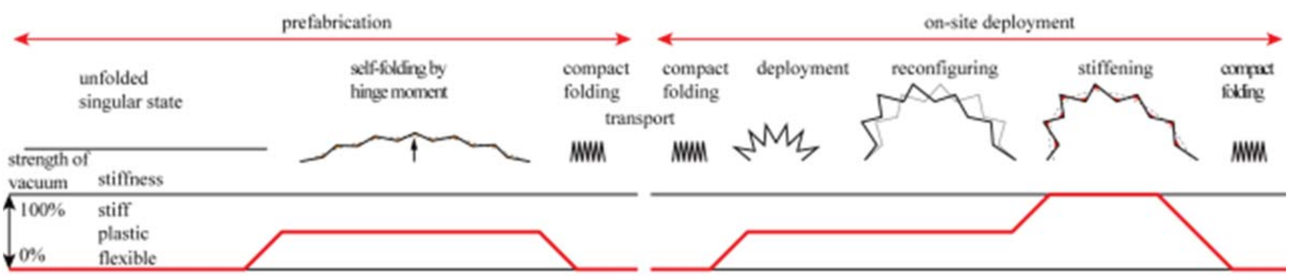

Figure 5: Idea of the construction process

3. Kinematics

3.1 Basics

The kinetic behavior of rigid origami based on triangle panels is generally described using unstable truss model. Here we denote the numbers of vertices, vertices on the boundary, and the edges by V, V_0, E. The configuration of the structure is represented by the coordinate $3V$-vector $\mathbf{x} = (\mathbf{x}_1 \quad \cdots \quad \mathbf{x}_V)^{\mathrm{T}} = (x_1 \quad y_1 \quad z_1 \cdots x_V \quad y_V \quad z_V)^{\mathrm{T}}$, which are constrained by its edge lengths $\mathbf{L}(\mathbf{x})$

(E-vector) being unchanged from the original state. When $3V > E = 0$, the infinitesimal transformation of the structure can be represented by the Jacobian matrix ($E \times 3V$ matrix) of the length vector.

$$\left[\frac{\partial \mathbf{L}}{\partial \mathbf{x}}\right]\{\Delta \mathbf{x}\} = \{\mathbf{0}\} \tag{1}$$

A nontrivial solution of Equation (1) gives the transformation mode, and the number of degrees of freedom of the structure (including rigid transformation of the whole structure) is the dimension of the solution space, which is $DOF = 3V - E + S$, where S is the number of degenerate constraints or the number of self-equilibrium state. In the case of triangular mesh homeomorphic to a disk, the numbers of elements are related by Euler's equation, which yields $DOF = V_0 + 3 + S$. By adding sufficient number of support conditions, e.g., pinning vertices to the ground, the structure becomes stable ($DOF = 0$).

3.2 Singularity at a Flat State

The initial unfolded state of a rigid origami structure has at least V singular conditions. This is because the elements of the $1 \pmod 3$-st columns of the Jacobean matrix are all 0, i.e., $\partial L/\partial z_i = 0$, where $L = \left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2$. For a developable origami surface, this degeneracy holds only when every fold is unfolded to a plane. As a result, the valid configuration space comprises $3V - E$-dimensional cells sharing one point (where everything is unfolded). This means that we need to choose the right transformation mode at the singular flat state in order to reach a desired state (Figure 6). The bending moment at each edge resulting from the vacuum produces the initial transformation mode that follows correct mountain and valley assignment and also convexity and concavity of vertices.

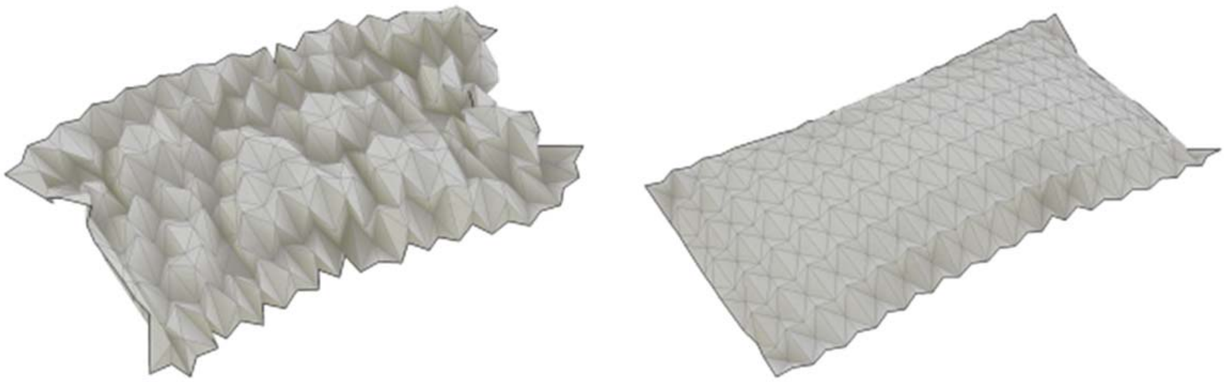

Figure 6: Unfavorable (Left) and valid (Right) transformation modes.

3.3 3D state and Constraints

In a generic half folded state, the degrees of freedom is given by $DOF = V_0 + 3$. This implies that we can fix the whole configuration by only fixing a part of the boundary points, for example pinning at least $V_0/3 + 1$ vertices. In Figure 7, we are pinning nearly every other boundary points to the ground, which results in an statically indeterminate stable structure with about $V_0/6$ excess constraints.

Therefore, the whole 3D configuration is dynamically controlled by the configuration of the pinned vertices. Because of the excess constraints, the configuration cannot be determined arbitrarily. Here, we show an interactive approach for formfinding the valid configuration of pinned vertices. We use the unstable truss model in Equation 1 and add geometric constraints (Figure 7),

1. Fold Angle Constraints: for every fold angle ρ, $|\rho| \leq 180° - \delta$.

This condition comes from the design of hinges for treating the thickness. $\delta = 11°$ is chosen in the example design.

2. Ground Constraints: for every vertex to be pinned on the ground plane, i.e., $\mathbf{n}^{\mathrm{T}}(\mathbf{x}_i - \mathbf{x}^0{}_i) = 0$, where $\mathbf{n}$ is the normal of the plane and $\mathbf{x}^0{}_i$ is a reference point on the ground, e.g., $\mathbf{n} = \mathbf{e}_z$ and $\mathbf{x}^0{}_i = \mathbf{0}$ for a horizontal plane. Since our method is based on iteration, this is adaptable to an arbitrary nonlinear terrain surface.
3. User Defined Constraints:
In this example, several boundary points are constrained to be on a vertical plane, derived from an existing building.

In this model, the pinned vertices are allowed to slide on the ground and only fixed in one dimension. This forms an underdetermined system, in which an infinitesimal valid motion is represented by

$$[\mathbf{C}]\{\Delta\mathbf{x}\} = \begin{bmatrix} \frac{\partial \mathbf{L}}{\partial \mathbf{x}} \\ \frac{\partial \mathbf{G}}{\partial \mathbf{x}} \end{bmatrix} \{\Delta\mathbf{x}\} = \{\mathbf{0}\}, \quad (2)$$

where $\mathbf{G} = \mathbf{0}$ represents the above mentioned constraints. Here, inequality conditions are treated as equality conditions if and only if they are not satisfied (penalty function). The minimal norm solution of this equation from an arbitrary estimation $\Delta\mathbf{x}_0$ is given using Moore-Penrose generalized inverse $[\mathbf{C}]^+$.

$$\{\Delta\mathbf{x}\} = \{\mathbf{I} - [\mathbf{C}]^+[\mathbf{C}]\}\{\Delta\mathbf{x}_0\}, \quad (3)$$

We use *Freeform Origami* [10] for interactively giving $\Delta\mathbf{x}_0$ while numerically solving the constraints. We have checked that the transformation path exists between the unfolded state, the compactly folded state, and the states shown in Figure 8, while keeping the fold angle constraints and ground constraints. The continuous transformation sweeps the pinned vertices to draw planar paths. Moving the pinned vertices along the paths actuates the 3D configuration of the model in the physical model.

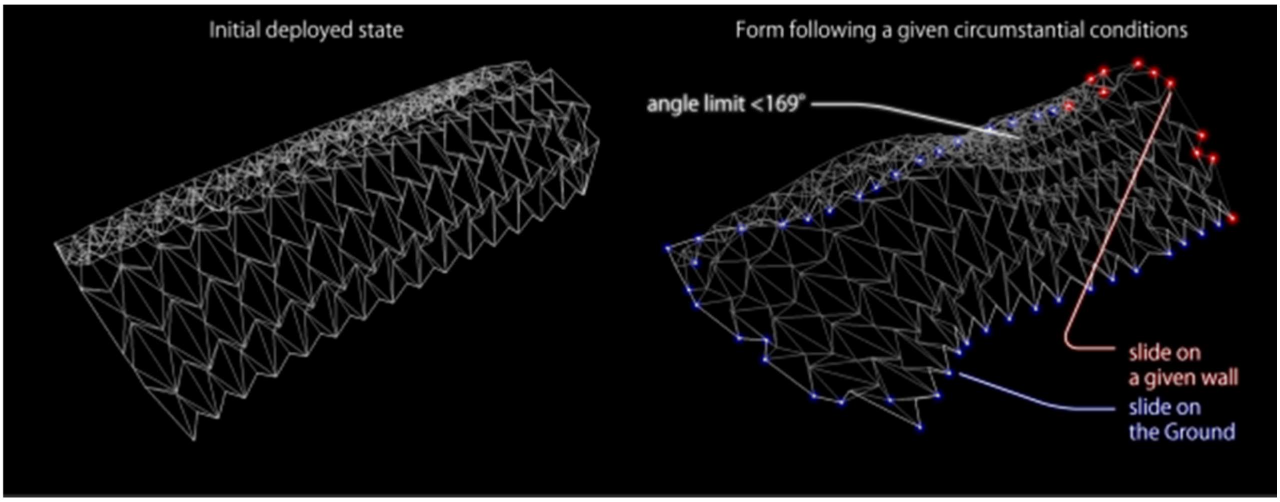

Figure 7: Geometric Constraints.

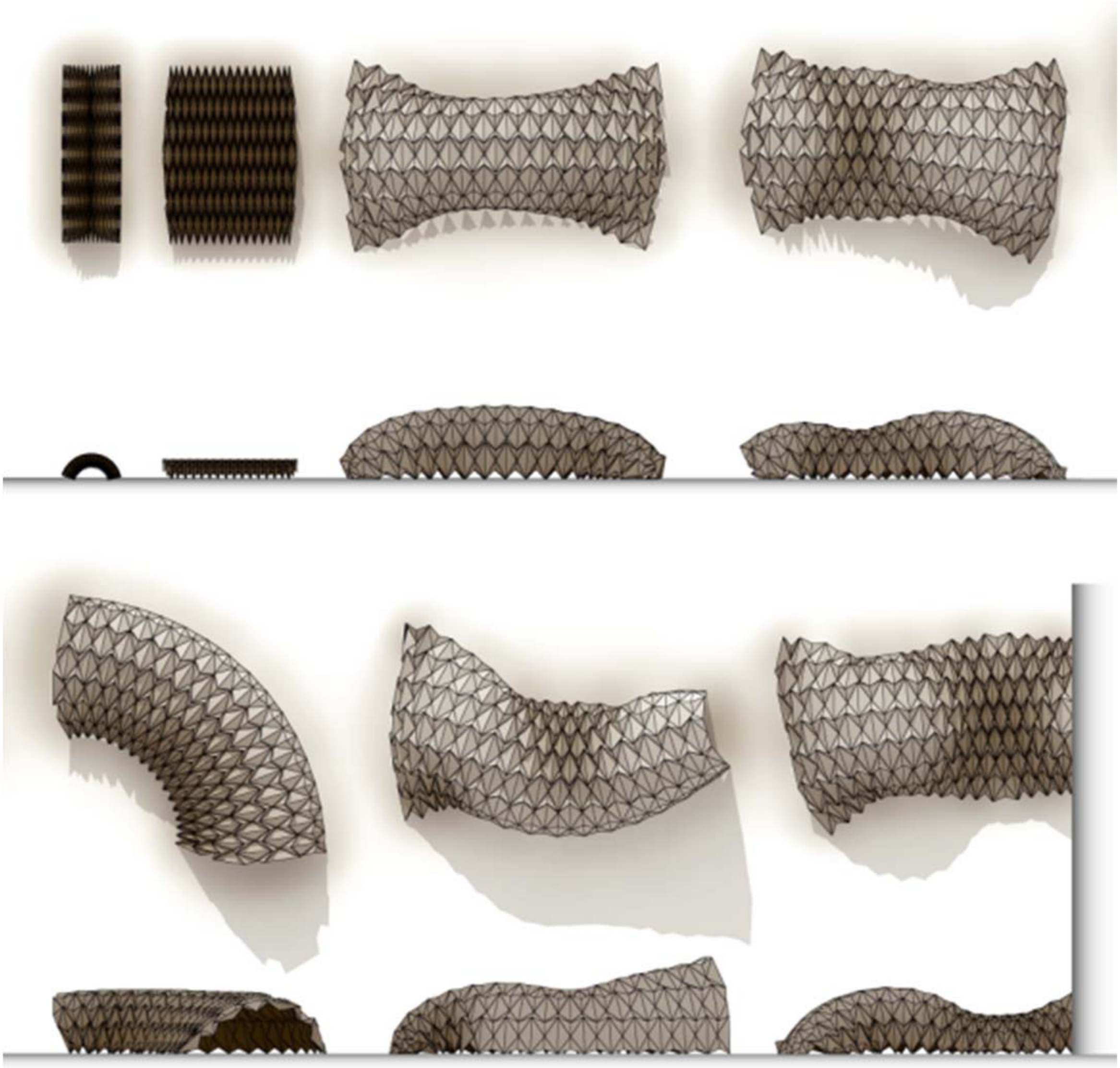

Figure 8: Variational configurations controlled by the boundary shapes.

Figure 9: Illustrative image of the example design with gourd shape.

4. Conclusion and Future works

We proposed a novel concept of hybrid structure based on multi-DOF rigid origami and Vacuumatics hinges in order to achieve an adaptive freeform surface whose 3D geometry and structural stiffness are controllable. By controlling the stiffness of the hinges, the stiffness of the structures can be effectively changed. The moment at the hinges produced by the vacuum removes the singularity in the unfolded state and makes the folding of the pattern controllable by the position of the boundary vertices.

In this paper, we only worked on the concept and the geometric behavior. We would like to develop the study to verify the effectiveness of the concept though experimental and analytical approaches. The patterns can be

5. Acknowledgement

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